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THE MONIST

THE PHILOSOPHY OF MR. B*TR*ND R*SS*LL.
WITH APPENDIXES OF LEADING PASSAGES FROM CERTAIN
OTHER WORKS.

"Even a joke should have some meaning."

The Red Queen, *T. L. G.*, p. 170.

[EDITOR'S NOTE.—When Mr. B*tr*nd R*ss*ll, following the advice of Mr. W*ll*m J*m*s, again got into touch with reality, and was torn to pieces by anti-suffragists, many of whom were political opponents of Mr. R*ss*ll, and held strict views on the necessity of protection, the following manuscript, which was almost ready for the press, was fortunately saved from the flames on the occasion when a body of eager champions of the Lost Cause of the Sacredness of Personal Property, from the city of Oxford, burnt the late Mr. R*ss*ll's house in B*gl*y W*d.]

ABBREVIATIONS:

- A. A. W.* Lewis Carroll, *Alice's Adventures in Wonderland*. London: Macmillan, 1905. People's Edition, 140th thousand.
- T. L. G.* Lewis Carroll, *Through the Looking-Glass, and What Alice Found There*. London: Macmillan, 1905. People's Edition, 87th thousand.
- S. B.* Lewis Carroll, *Sylvie and Bruno*. London: Macmillan, 1889.
- Ph. L.* Bertrand Russell, *A Critical Exposition of the Philosophy of Leibniz, with an Appendix of Leading Passages*. Cambridge: University Press, 1900.
- Pr. M.* Bertrand Russell, *The Principles of Mathematics*, Vol. I. Cambridge: University Press, 1903.
- A. d. L.* Ernst Schröder, *Vorlesungen über die Algebra der Logik*. Leipzig: Teubner, Vol. I, 1890; Vol. II, 1891 and 1905.
- Gg.* G. Frege, *Grundgesetze der Arithmetik*. Jena: Hermann Pohle, Vol. I, 1893; Vol. II, 1903.
- Z. S.* G. Frege, *Ueber die Zahlen des Herrn H. Schubert*. Jena, 1899.
- Gl.* G. Frege, *Die Grundlagen der Arithmetik, eine logisch-mathematische Untersuchung über den Begriff der Zahl*. Breslau, 1884.
- R. M. M.* *Revue de Métaphysique et de Morale*.
- S. L.* John Venn, *Symbolic Logic*. London: Macmillan, 1881; 2d ed., 1894.
- F. L.* Augustus De Morgan, *Formal Logic; or The Calculus of Inference, Necessary and Probable*. London, 1847.

- Fm. L.* John Neville Keynes, *Studies and Exercises in Formal Logic*. 4th ed., London, 1906.
- E. u. I.* Ernst Mach, *Erkenntnis und Irrtum. Skizzen zur Psychologie der Forschung*. 2d ed., Leipsic, 1906.
- G. u. E.* G. Heymans, *Die Gesetze und Elemente des wissenschaftlichen Denkens*. Leiden, Vol. I, 1890; Vol. II, 1894.
- A. C. P.* *The Annotated Book of Common Prayer*, ed. by John Henry Blunt, D.D. New Edition. London: Rivingtons, 1888.

THE INDEFINABLES OF LOGIC.

THE view that the fundamental principles of logic consist solely of the law of identity was held by Leibniz,¹ Drobisch, Ueberweg,² and Tweedledee.³ If this were the case, the principles of logic could hardly be said to be, as they are, a body of propositions whose consistency is impossible to prove.⁴ This characteristic is important and one of the marks of the greatest possible security; for while a great achievement of late years has been to prove the consistency of the principles of arithmetic, a science which is unreservedly accepted except by some empiricists,⁵ it can be proved formally that one foundation of arithmetic is shattered.⁶ It is true that it has been shown quite lately that this conclusion may be avoided, and by a re-moulding of logic we can draw instead the paradoxical conclusion that the opinions held by common sense for so many years are in part justified; but it is quite certain that with the principles of logic no such proof of consistency and no such paradoxical result of further investigations are to be feared.

Still, this re-moulding has had the result of bringing logic into tolerable agreement with common sense. There

¹ Russell, *Ph. L.*, pp. 17, 19, 207-208.

² Schröder, *A. d. L.*, I, p. 4.

³ See Appendix A.

⁴ Cf. Pieri in *R. M. M.*, March, 1906, p. 199.

⁵ As a type of these, Humpty-Dumpty, with his inability to admit anything not empirically given, and his lack of comprehension of pure mathematics, may be taken (See Appendix B). In his (correct) thesis that definitions are nominal, too, Humpty-Dumpty reminds one of J. S. Mill (see Appendix C).

⁶ See Frege, *Gg.*, II, p. 253.

were only two alternatives: If we chose principles in accordance with common sense we arrived at conclusions which shocked common sense; by starting with paradoxical principles, we have arrived at ordinary conclusions. Like the White Knight,⁷ we have dyed our whiskers an unusual color and then hidden them.

The quaint name of the "Laws of Thought" which is often applied to the principles of logic, has given rise to confusion in two ways: In the first place the "Laws," unlike other laws, cannot be broken, even by refusing to think; and in the second place people think that the laws have something to do with holding for the operations of their minds, just as laws of nature hold for events in the world around us.⁸ But that the laws are not psychological laws follows from the facts that a thing may be true even if nobody believes it, and something else may be false if everybody believes it. Indeed it generally is.

Fortunately, the principles or laws of logic are not a matter of philosophical discussion. Idealists like Tweedledum and Tweedledee, and even practical idealists like the White Knight, explicitly accept laws like the law of identity and the excluded middle, as we have seen above or shall see in the Appendix, under E.

In fact, throughout all logic and mathematics, the existence of the human or any other mind is totally irrelevant; mental processes are studied by means of logic, but the subject-matter of logic does not presuppose mental processes, and would be equally true if there were no mental processes, It is true that in that case we should not know logic; but our knowledge must not be confounded with the truths which we know, any more than an apple should be with the eating of it.⁹

⁷ See Appendix, D.

⁸ See Frege, *Gg.* I, p. xv.

⁹ B. Russell, *Hibbert Journal*, July, 1904, p. 812.

IDENTITY.

Identities are frequently used in common life by people who seem to imagine that they can draw important conclusions respecting conduct or matters of fact from them. I have heard of a man who gained the double reputation of being a philosopher and a fatalist by the repeated enunciation of the identity, "Whatever will be, will be"; and the Italian equivalent of this makes up an appreciable part of one of Mr. Robert Hichens's novels. Further, the identity "life is life" has not only been often accepted as an explanation for a particular way of living, but has even been considered by an authoress who calls herself "Zack" to be an appropriate title for a novel; while "business is business" is frequently thought to provide an excuse for dishonesty in trading, for which purpose it is plainly inadequate.

Another example is given by a poem of Mr. Kipling's, where he seems to assert that "East is East" and "West is West" imply that "never the twain shall meet." The conclusion, now, is false; for, since the world is round—as geography books still maintain by arguments which strike every intelligent child as invalid¹⁰—what is called the "West" does, in fact, merge into the "East." Even if we are to take the statement metaphorically it is still untrue, as the Japanese nation have shown.

The law courts are often rightly blamed for being strenuous opponents of the spread of symbolic logic; the frequent misuse of *and*, *or*, *the*, and *provided that* in them is notorious. But the fault seems partly to lie in the uncomplicated nature of the logical problems which are dealt

¹⁰ The argument of the hull of a ship disappearing first is not convincing, since it would equally well prove that the surface of the earth was, for example, corrugated on a large scale. If the common sense of the reader were supposed to dismiss the possibility of water clinging to such corrugations, it might equally be supposed to dismiss the possibility of water clinging to a spherical earth. Traditional geography books, no doubt, gave rise to the opinions held by Lady Blount and the Zetetic Society.

with in them. Thus it is no uncommon thing for people to appear there who are unable to establish their own identity, or for A to assert there that B was not himself when he made a will leaving his money to C.

The chief use of identities is in implication. Since, in logic, we so understand *implication* that any true proposition implies and is implied by any other true proposition, if one is convinced of the truth of the proposition Q, it is advisable to choose one or more identities (P), whose truth is undoubted, and say that P implies Q. Thus Mr. Austen Chamberlain, according to the *Times* of March 27, 1909, professed to deduce the conclusion that it is not right that women should have votes from the premises that "man is man" and "woman is woman." Unfortunately this method requires that one should have made up one's mind about the conclusion before discovering the premises—by what, no doubt, Jevons would call an inverse or inductive method. Thus the method is only of use in speeches.

Mr. Austen Chamberlain afterwards rather destroyed one's belief in the truth of his premises, by putting limits to the validity of the principle of identity. In the course of the debate on the Budget of 1909, he maintained, against Mr. Lloyd George, that a joke was a joke except when it was an untruth, Mr. Lloyd George, apparently, being of the opinion that a joke is a joke under all circumstances.

SYMBOLISM AND MEANING, AND SIGN AND SIGNIFICATION.

When people write down any statement such as "The curfew tolls the knell of parting day," which we will call "C" for shortness, what they mean is not C but the *meaning* of C; and not "the meaning of C" but the *meaning* of "the meaning of C." And so on *ad infinitum*. Thus in writing or in speech we always fail to state the meaning of any proposition whatever. Sometimes, indeed, we succeed in *conveying* it; but there is danger in too great a

disregard of statement and preoccupation with the conveyance of meaning. Thus many mathematicians have been so anxious to convey to us a perfectly distinct un-metaphysical concept of number, that they stripped away everything that they considered unessential (like its logical nature) from the idea of number, and have finally delivered it to us as a mere *sign*. By the labor of Helmholtz, Kronecker, Heine, Thomae, Pringsheim and Schubert, many people were persuaded that when they said "2 is a number" they were speaking the truth, and hold that "Paris" is a town containing a *p*.¹¹ When Frege pointed out this difficulty, e. g., in *Z. S.*, he was almost universally denounced as "*spitzfindig*." In fact, Germans seem to have been influenced by Kant to despise the White Knight's subtle distinctions¹² and to regard subtlety with disfavor to such a degree that their only mathematical logician except Frege, namely Schröder—the least subtle of mortals, by the way—seems to have been filled with such fear of being thought subtle, that he made his books so prolix that nobody has read them.

Another term which mathematicians are accustomed to apply to thought which is more exact than any to which they are accustomed is "scholastic." Thereby, I suppose, they mean that the pursuits of certain acute people of the Middle Ages are unimportant as compared with the great achievements of modern thought, as exemplified by a method of making plausible guesses, known as induction; by the bicycle and the gramophone—all of them instruments of doubtful merits.

¹¹ De Morgan (*F. L.*, pp. 246-247) said that "if all mankind had spoken one language, we cannot doubt that there would have been a powerful, perhaps universal, school of philosophers who would have believed in the inherent connection between names and things; who would have taken the sound *man* to be the mode of agitating the air which is essentially communicative of the ideas of reason, cookery, bipedality, etc., . . . 'The French,' said the sailor, 'call a cabbage a *shoe*;' the fools! Why can't they call it a cabbage when they must know it is one?"

¹² See Appendix E.

PREVIOUS PHILOSOPHICAL THEORIES OF MATHEMATICS BY
MATHEMATICIANS.

Mathematicians usually try to found mathematics on two principles. One is the principle of confusion between the sign and the thing signified (they call this principle the foundation-stone of the formal theory), and the other is the principle of the identity of discernibles (which they call the principle of the permanence of equivalent forms).¹³

But the truth is that if we set sail on a voyage of discovery with logic alone at the helm, we must either throw such principles as "the identity of those conceptions which have in common the properties that interest us" and "the principle of permanence" overboard, or, if we do not like to act in such a way to old companions with whom we are so familiar that we can hardly feel contempt for them, we must at least recognize them clearly as having no logical validity and merely as psychological principles, and reduce them to the humble rank of stewards to minister to our human weaknesses on the voyage. And then, if we adopt the wise policy of keeping our axioms down to the minimum number, we must refrain from creating, or perhaps rather thinking we can create, new numbers to fill up gaps among the older ones, and then recognize that our rational numbers are particular cases of "real" numbers, and so on.

We get, by this, a world of conceptions which looks, and is, different from that which ordinary mathematicians think they see; and perhaps this is the reason why some mathematicians of great eminence, like Hilbert and Poincaré,¹⁴ have produced such absurd discussions on the fundamental principles of mathematics, showing once more

¹³ These principles, after many attempts to state them by Peacock, the Red and the White Queen (see Appendix G), Hankel, Schröder, and Schubert had been made, were first exactly formulated by Frege in *Z. S.*

¹⁴ See Couturat, *R. M. M.*, March, 1906, and Russell, *ibid.*, Sept. 1906.

the truth of the not quite original remark of Aunt Jane, who

".....observed, the second time
She tumbled off a 'bus:
'The step is short from the sublime
To the ridiculous.'"

AMBIGUITY AND SYMBOLIC LOGIC.

The universal use of some system of symbolic logic would not only enable everybody easily to deal with exceedingly complicated arguments, but would prevent ambiguous statements. In denying the indispensability of symbolic logic in the former state of things, Dr. Keynes (*Fm. L.*) is probably alone,¹⁵ against the need strongly felt by Alice and most modern logicians. (See Appendix H).

As regards ambiguity, a translation of *Hymns Ancient and Modern* into, say, Peanese, would prevent the well-known puzzle of childhood as to whether the "his" in

"And Satan trembles when he sees
The weakest saint upon his knees,"

refers to the saint's knees or Satan's.

ASSERTION.

The subject of the present chapter must not be confused with the assertions of ordinary life. Commonly an unasserted proposition is synonymous with a probably false statement, while an asserted proposition is synonymous with one that is certainly false. But in logic we apply assertion also to true propositions and, as Lewis Carroll showed in his version of "What the Tortoise said to Achilles,"¹⁶ usually pass over unconsciously an infinite series of implications in so doing. If p and q are propositions,

¹⁵ The Duchess is more consistent than Keynes, for Keynes really uses the \times and $+$ of Boole and Venn under the different shapes of the words "and" and "or."

¹⁶ *Mind*, New Series, Vol. IV, 1895, pp. 278-280. Cf. Russell, *Pr. M.*, p. 35.

p is true and p implies q ; then, at first sight, one would think that one might assert q . For, from (A) " p is true," and (B) " p implies q ," we must, in order to deduce (Ω) " q is true," accept the hypothetical (C) "If A and B are true, Ω must be true." And then, in order to deduce Ω from A, B, C, we must accept another hypothetical (D) "If A, B, and C are true, Ω must be true"; and so on *ad infinitum*. Thus, in deducing Ω , we pass over an infinite series of hypotheticals which increase in complexity. Thus we need a new principle to be able to assert q .

Frege was the first logician sharply to distinguish between an asserted proposition, like "A is greater than B," and one which is merely considered, like "A's being greater than B," although an analogous distinction had been made in our common discourse, on certain psychological grounds, for long previously. In fact, soon after the invention of speech, the necessity of distinguishing between a considered proposition and an asserted one became evident, on account of the state of things referred to at the beginning of this chapter.

IS.

Is has four perfectly distinct meanings in English, besides misuses of the word. Among the misuses, perhaps the most important are those referred to by De Morgan:¹⁷ "...we say 'murder *is* death to the perpetrator' where the copula is *brings*; 'two and two *are* four,' the copula being 'have the value of,' etc."

Schröder¹⁸ quite satisfactorily pointed out the well-known distinction between an *is* where subject and predicate can be interchanged (such as: "the class whose members are Shem, Ham, and Japhet is the class of the sons of Noah") and an *is* or *are* where they cannot (such as:

¹⁷ *F. L.*, p. 268.

¹⁸ *A. d. L.*, I.

"Englishmen are Britons"), but failed to see¹⁹ the more important distinction (made by Peano) of *is* in the sense of "is a member of." If Englishmen are Britons, and Britons are civilized people, it follows that Englishmen are civilized people; but though the *Harmsworth Encyclopaedia* is a member of the class "books (of one or more volumes)," and this class is the member of some class A of which it is the only member, yet the *Harmsworth Encyclopaedia* is not a member of A, for it is not true that it is the whole class of books; and such a statement would not even be made, except possibly in the form of an advertisement.

The fourth meaning of *is* is *exists*; it is a matter for regret that there are difficulties in the way of using one word to denote four things with different meanings; for, if there were not, we might prove the existence of Anything by making It the subject of a proposition, and thus earn the gratitude of theologians.

"AND" AND "OR."

When, with Boole, alternatives (A, B) are considered as mutually exclusive, logical addition may be described as the process of taking A *and* B or A *or* B. It is a great and rare convenience to have two terms for denoting the same thing: commonly, people denote several things by the same term, and only the Germans have the privilege of referring to, say, continuity as *Stetigkeit* or *Kontinuirlichkeit*. But Jevons²⁰ quoted Milton, Shakespeare, and Darwin to prove that alternatives are not exclusive, and so attained first to recognized views by an argument which was plainly inadequate for his purpose.

Of course, "and" is often used as the sign of logical addition: thus one may speak of one's brothers and sisters,

¹⁹ *Ibid.*, II.

²⁰ *Pure Logic*, . . . , London, 1864, pp. 76-79. Cf. Venn, *S. L.*, 2d ed., pp. 40-48.

without being understood to mean the null-class (as should be the case).²¹ And a word like "while" is often used for a logical addition, when exclusiveness of the alternatives is almost implied. Thus, a reviewer in *Mind*,²² noticing the translation of Mach's *Popular Science Lectures* into American, said, of these lectures, that: "Most of them will be familiar . . . to epistemologists and experimental psychologists; while the remainder, which deal with physical questions, are well worth reading." The reader has the impression, probably given unintentionally, that Professor Mach's epistemological and psychological lectures are not, in the reviewer's opinion, worth reading.

THE COMMUTATIVE LAW.

Often the meaning of a sentence tacitly implies that the commutative law does not hold. We are all familiar with the passage in which Macaulay pointed out that by using the commutative law because of exigencies of meter, Robert Montgomery unintentionally made Creation tremble at the Atheist's nod instead of the Almighty's. This use of the commutative law by writers of verse renders it doubtful whether, in the hymn-line:

"The humble poor believe,"

we are to understand a statement about the humble poor, or a doubtful maxim as to the attitude of our minds to statements made by the humble poor.

Then non-commutativity to English titles offers difficulties to some novelists and Americans, who make a point of referring to Mary Lady So-and-So as Lady Mary, and *vice versa*.

²¹ Children sometimes pray for their relations *and* friends; two plainly exclusive classes.

²² New Series, IV, p. 261.

THE.

The word "the" implies existence and uniqueness. It is a mistake to talk of "*the* son of So-and-So" if So-and-So has a fine family of ten sons. People who refer to "the Oxford Movement" imply that Oxford only moved once; and those quaint people who say that "A is quite the gentleman" imply both the doubtful proposition that there is only one gentleman in the world, and the indubitably false proposition that he is that man. Probably A is one of those persons who add to the confusion in the use of the definite article by speaking of his wife as "the wife."

In a certain children's hymnbook, one reads:

"The river vast and small."

Few would deny that there is not more than one such river, but unfortunately it is doubtful if there is such a river at all. The case is exactly the same with the ontological proof of the existence of the most perfect being.

According to the *Daily Mail* of October 9, 1906, Judge Russell decided against a claim brought by an agent against his company for appointing another agent, the claim being on the grounds that he was appointed as "the" agent.

Most people admit that the number 2 can be added to the number 2 to give the number 4, but this is a mistake. They concede, when they use *the*, that there is only one number 2, and yet they imagine that, when they remove this, to consider it apart as the first term of our above sum, they can find another to add to it, and thereby form the second term. The truth is, that " $2+2=4$ " is a very misleading equation, and what we really mean by that faultily abbreviated statement is: If x and y denote any things, and x' and y' any other things, which form a class (A) which, like that of x and y , is a member of that class (which we call "2"), of classes which have a correspond-

ence with what we call a class B of two things, such that, if any member of A corresponds to one, and only one, member of B, and inversely; for the class of all the terms x, y, x', y' is a member of that class of classes which, analogously, we call "4." In this, for the sake of shortness, we have introduced abbreviations which should not be used in a rigorous logical statement.

UNIVERSAL AND PARTICULAR PROPOSITIONS.

People who are cynical as to the morality of the English are often unpleasantly surprised to learn that "All trespassers will be prosecuted" does not necessarily imply that "Some trespassers will be prosecuted." The view that universal propositions are non-existential is now generally held. Venn seems to have been the first to hold this, while older logicians, such as De Morgan,²³ considered universal propositions to be existential, like particular ones.

If the Gnat²⁴ had been content to affirm his proposition about the means of subsistence of a Bread-and-butter-fly, in consequence of their lack of which such flies always die, without pointing out such an insect, and thereby proving that the class of them is not null, Alice's doubt as to the existence of the class in question, even if it were proved to be well-founded, would not have affected the validity of the proposition.

This brings us to a great convenience in treating universal propositions as non-existential. We can maintain that all x 's are y 's at the same time as that no x 's are y 's, if only x is the null-class. Thus when Mr. MacColl²⁵ objected to other symbolic logicians that their premises imply that all Centaurs are flower-pots, they could reply that

²³ Cf. *F. L.*, p. 4.

²⁴ See Appendix I.

²⁵ Cf. *Mind*, 1905.

their premises also imply the more usual view that Centaurs are not flower-pots.

IMPLICATION.

A good illustration of the principle that what we call "implication" in logic is such that a false proposition implies any other proposition, true or false, is given by Lewis Carroll's puzzle of the three barbers.²⁸

Allen, Brown, and Carr keep a barber's shop together; so that one of them must be in during working hours. Allen has lately had an illness of such a nature that, if Allen is out, Brown must be accompanying him. Further, if Carr is out, then, if Allen is out, Brown must be in for obvious business reasons. The problem is, may Carr ever go out?

Putting p for "Carr is out," q for "Allen is out," and r for "Brown is out," we have:

- (1) q implies r ,
- (2) p implies that q implies not- r .

Lewis Carroll supposed that " q implies r " and " q implies not- r " are inconsistent, and hence that p must be false. But both these propositions are true if q is false. Thus, if p is true, q is false; or, if Carr is out, Allen is in. The odd part of this conclusion is that it is the one which common sense would have drawn in that particular case.

The principle that the false implies the true has very important applications in political arguments. In fact, it is hard to find one principle of politics of which false propositions are not the main support.

If p and q are two propositions, and p implies q ; then, if, and only if, q and p are both false or both true, we also have " q implies p ." The most important applications of

²⁸ *Mind*, N. S., III, 1894, pp. 436-438. Cf. the discussions by W. E. Johnson, *ibid.*, p. 583, and Russell, *Pr. M.*, p. 18n, and *Mind*, N. S., XIV, 1905, pp. 400-401.

this invertibility were made by the late Mr. Samuel Butler²⁷ and by Mr. G. B. Shaw. A political application may be made as follows. In a country where only those with middling-sized incomes are taxed, conservative and *bourgeois* politicians would still maintain that the proposition "the rich are taxed" implies the proposition "the poor are taxed," and this implication—which is true because both protasis and apodasis are false—would be quite unnecessarily supported by many false practical arguments. It is equally true that "the poor are taxed" implies that "the rich are taxed." And this can be proved in certain cases on other grounds. For the taxation of the poor would imply, ultimately, that the poor could not afford to pay a little more for the necessities of life than, in strict justice, they ought; and this would mean the cessation of one of the chief means of production of individual wealth.

We also see why a valuable means for the discovery of truth is given by the inversion of platitudinous implications. It may happen that another platitude is the result of inversion; but it is the fate of any true remark, especially if it is easy to remember by reason of a paradoxical form, to become a platitude in course of time. There are rare cases of a platitude remaining unpeated for so long that, by a converse process, it has become paradoxical. Such, for example, is Plato's remark that a lie is less important than an error in thought.

Of late years, a method of disguising platitudes as paradoxes has been too extensively used by Mr. G. K. Chesterton. The method is as follows. Take any proposition p which holds of an entity a ; choose p so that it seems plausible that p also holds of at least two other entities b and c ; call a , b , c , and any others for which p holds or seems to hold, the class A , and p the A -ness or A -ity of a , b , and c ;

²⁷ The inhabitants of "Erewhon" punished invalids more severely than criminals. In modern times, one frequently hears the statement that crime is a disease; and if so, it is surely false that criminals ought to be punished.

let d be an entity for which p does not hold; and put d among the A 's when you think that nobody is looking. Then state your paradox: "Some A 's do not have A -ness." By further manipulation you can get the proposition "All A 's do not have A -ness." But it is possible to make a very successful *coup* if A is the null-class, which has the advantage that manipulation is unnecessary. Thus, Mr. Chesterton, in his *Orthodoxy*, put $A =$ the class of doubters who doubt the possibility of logic, and proved that such agnostics refuted themselves—a conclusion which seems to have pleased many clergymen.

In this way, Mr. Chesterton has been enabled readily to write many books, and to maintain, on almost every page, such theses as that simplicity is not simple, heterodoxy is not heterodox, poetry is not poetical, and so on; thereby building up the gigantic platitude that Mr. Chesterton is Chestertonian.

In the chapter on "Identity" we have illustrated the use of the principle that any true proposition implies any other true proposition. This important principle may be called *the principle of the irrelevant premise*;²⁸ and is of great service in oratory because it does not matter what the premise is, true or false. There is a *principle of the irrelevant conclusion*, but, except in law courts, in interruptions of meetings, and in family life, this is seldom used, partly because of the limitation involved in the logical impossibility for the conclusion to be false if the premise be true, but chiefly because the conclusion is more important than the premise, being usually a matter of prejudice.

Certain modern logicians, such as Frege, have found it necessary so to extend the meaning of implication of q by p that it holds when p is not a proposition at all. Hith-

²⁸ *Irrelevant* in a popular sense; one would say, speaking loosely, that the fact that Brutus killed Cæsar is irrelevant to the fact that the sea is salt; and yet this conclusion is implied both by the above premise and the premise that Cæsar killed Brutus. Cf. on such questions, Venn, *S. L.*, 2d ed., pp. 240-244.

erto, politicians, finding that either identical or false propositions are sufficient for their present needs, have made no use of this principle; but it is obvious that their stock of arguments would be vastly increased thereby.

Logical implication is often an enemy of dignity and eloquence. De Morgan²⁹ relates "a tradition of a Cambridge professor who was once asked in a mathematical discussion 'I suppose you will admit that the whole is greater than its part?' and who answered, 'Not I, until I see what use you are going to make of it.' " And the care displayed by cautious mathematicians like Poincaré, Schoenflies, Borel, Hobson, and Baire in abstaining from pushing their arguments to their logical conclusions is probably founded on the unconscious—but no less well-grounded—fear of appearing ridiculous if they dealt with such extreme cases as "the series of all ordinal numbers." They are, probably, as unconscious of implications as the author of the remark that Gibbon always had a copy of Horace in his pocket and often in his hand, was of the necessary implication of these propositions that Gibbon's hand was sometimes in his pocket.

DENOTING.³⁰

A concept *denotes* when, if it occurs in a proposition, the proposition is not *about* the concept, but about a term connected in a certain peculiar way with the concept. Some people often assert that man is mortal, and yet we never see announced in the *Times* that Man died on a certain day at his villa residence "Camelot" at Upper Tooting; nor do we hear that Procrastination was again the butt of Mr. Plowden's jokes at Marylebone Police Court last week.

That two phrases may have different *meanings* and the same *denotation* was discovered by Alice³¹ and Frege.

²⁹ *F. L.*, p. 264.

³⁰ Cf. Russell, *Pr. M.*, pp. 53-54.

³¹ See Appendix J.

Alice observed that the road which led to Tweedledum's house was that which led to the house of Tweedledee; and Frege pointed out that the phrases "the house to which the road that leads to Tweedledum's house," and "the house to which the road that leads to Tweedledee's house" have different *Sinn* but the same *Bedeutung*.

NON-ENTITY.

When people say that such-and-such a thing "is non-existent," they usually mean that it is not an *it* at all, or that there is not any it.

Dr. Venn meant this when he described (in *S. L.*, 1881, p. 339n) his encounter with what he imagined to be a very ingenious tradesman: "I once had some strawberry plants furnished me which the vendor admitted would not bear many berries. But he assured me that this did not matter, since they made up in their size what they lost in their number. (He gave me, in fact, the hyperbolic formula, $xy=c^2$, to connect the number and magnitude). When summer came *no* fruit whatever appeared. I saw that it would be no use to complain, because the man would urge that the size of the non-existent berry was infinite, which I could not see my way to disprove. I had forgotten to bar zero values of either variable."

It is to be regretted that this useful note was omitted in the second edition of *S. L.*; one can imagine that it might have protected Mr. MacColl and Herr Meinong (who believed in round squares and fabulous monsters),³² against the dishonest practices of traders who were too free in their promises. For the death-blow to this kind of free trade was not given until 1905, when Mr. Russell published his article "On Denoting," and took up the position of the White King in opposition to Alice's later assertions.³³

³² This belief was unlike Alice's first opinion (see Appendix K).

³³ See Appendix K.

Venn's experience illustrates another characteristic of mathematical logic. It is necessary, in order to make our arguments conclusive, to devote great care to the elimination of difficulties which rarely occur. The White Knight—who was like Boole in being a pioneer of mathematical logic in this way, and seems to have held, like Boole, those philosophical opinions which would base logic on psychology—recognized the necessity of taking precautions against any unusual appearance of mice on a horse's back.³⁴

THE UNKNOWABLE.

According to Mr. S. N. Gupta,³⁵ the first thing that every student of Hindu logic has to learn when he is said to begin the study of inference is that "all H is S" is not always equivalent to "no H is not S." "The latter proposition is an absurdity when S is *Kebalánvayi*, i. e., covers the whole sphere of thought and existence. . . . 'Knowable' and 'Nameable' are among the examples of *Kebalánvayi* terms. If you say there is a thing not-knowable, how do you know it? If you say there is a thing not-nameable, you must point that out, i. e., somehow name it. Thus you contradict yourself."

Mr. Herbert Spencer's doctrine of the Unknowable gives rise to some amusing thoughts. To state that all knowledge of such and such a thing is above a certain person's intelligence is not self-contradictory, but merely rude; to state that all knowledge of a certain thing is above all possible human intelligence is, in spite of its appearing to be a modest platitude, nonsense. For the statement shows that we do know something of it, viz., that it is unknowable.

It is somewhat amusing to find that to the last (1900) edition of *First Principles* was added a "Postscript to Part

³⁴ See Appendix L.

³⁵ *Mind*, N. S., IV, 1895, p. 168.

I,"³⁶ in which the justice of this simple and well-known criticism as to the contradiction involved in speaking of an "Unknowable," which had been often made during the forty odd years in which the various editions had been on the market, was grudgingly acknowledged as follows:

"It is doubtless true that saying what a thing is not, is, in some measure, saying what it is; . . . Hence it cannot be denied that to affirm of the Ultimate Reality that it is unknowable is, in a remote way, to assert some knowledge of it, and therefore involves a contradiction."

The "Postscript" reminds one of the postscript to a certain Irishman's letter. This Irishman, missing his razors after his return from a visit to a friend, wrote to his friend, giving precise directions where to look for the missing razors; but, before posting the letter, added a postscript to the effect that he had found the razors.

One is tempted to inquire, analogously, what might be, in view of the Postscript, the point of much of Spencer's Part I. It is, to use De Morgan's description of the arguments of some who maintain that we can know nothing about infinity,³⁷ of the same force as that of the man who answered the question how long he had been deaf and dumb.

The analogy of the contradiction of Burali-Forti to the contradiction involved in the notion of an "unknowable" may be set forth as follows. If A should say to B: "I know things which you never by any possibility can know," he may be speaking the truth. In the same way, infinity may be said, without contradiction, to transcend all the *finite* integers. But if some one else, C, should say: "There are some things which no human being can ever know any-

³⁶ *First Principles*, 6th ed., 1900, pp. 107-110. The first edition was published in 1862.

³⁷ Note on p. 6 of his paper: "On Infinity; and on the Sign of Equality," *Trans. Camb. Phil. Soc.*, XI, Part I, pp. 1-45. (Read May 16, 1864.)

thing about," he is talking nonsense.³⁸ And in the same way if we succeed in imagining a number which transcends *all* numbers, we have succeeded in imagining the absurdity of a number which transcends itself.

All the paradoxes of logic (or "the theory of aggregates") are analogous³⁹ to the difficulty arising from a man's statement: "I am lying." In fact, if this is true, it is false, and *vice versa*. If such a statement is spread out a little, it becomes an amusing hoax or an epigram. Thus, one may present to a friend a card bearing on both sides the words: "The statement on the other side of this card is false;" while the first of the epigrams derived from this principle seems to have been written by a Greek satirist:⁴⁰

"Lerians are bad: not *some* bad and some *not*;
But all. There's not a Lerian in the lot,
Save Procles, that you could a good man call—
And Procles is a Lerian after all."

This is the original of a well-known epigram by Porson, who remarked that all Germans are ignorant of Greek meters,

"All, save only Hermann—
And Hermann's a German."

MR. SPENCER, THE ATHANASIAN CREED, AND THE ARTICLES.

When, in what I believe is misleadingly known as "The Athanasian Creed," people say "The Father incomprehensible," and so on, they are not falling into the same error as Mr. Spencer, for the Latin equivalent for "incomprehensible" is merely *immensus*,⁴¹ and Bishop Hilsey translated it more correctly as "immeasurable." It is a regrettable

³⁸ I think that all the talk about the finitude of man's mind is nonsense; both because, if we say that the mind of man is limited, we tacitly postulate an 'unknowable' and because, even if the human mind were finite, there is no more reason against its conceiving the infinite than there is for a mind to be blue in order to conceive of a pair of blue eyes (Cf. De Morgan, *loc. cit.*).

³⁹ Russell, *R. M. M.*, Sept. 1906.

⁴⁰ *The Greek Anthology*, by Lord Neaves (Ancient Classics for English Readers). Edinburgh and London, 1897, p. 194.

⁴¹ *A. C. P.*, p. 217.

fact that Dr. Blunt, in his mistaken modesty, has added a note⁴² to this passage: "Yet it is true that a meaning not intended in the Creed has developed itself through this change of language, for the nature of God is as far beyond the grasp of the mind as it is beyond the possibility of being contained within local bounds."

Mr. Spencer seems no happier when we compare his statements with those in the Anglican Articles of Religion. There God is never referred to as infinite. It is true that his power and goodness are so referred to; but this deficiency was presumably brought about intentionally, so that faith might gain in meaning as time went on.

"GEDANKENEXPERIMENTE" AND EVOLUTIONARY ETHICS.

The "*Gedankenexperimente*," upon which so much weight has been laid by Mach⁴³ and Heymanns,⁴⁴ had already been investigated by the White Queen,⁴⁵ who, however, seems to have perceived that the results of such experiments are not always logically valid. The psychological founding of logic appears to be not without analogy with the surprising method of advocates of evolutionary ethics who expect to discover what is good by inquiring what cannibals have *thought* good. I sometimes feel inclined to apply the historical method to the multiplication table. I should get a statistical inquiry among school-children, before their pristine wisdom had been biased by teachers. I should put down their answers as to what 6×9 amounts to; I should work out the average of their answers to six places of decimals, and should then decide that, at the present stage of human development, this average is the value of 6×9 .

⁴² *Ibid.*, p. 218.

⁴³ See, e. g., *E. u. I.*, pp. 183-200.

⁴⁴ *G. u. E.*, Vol. I.

⁴⁵ See Appendix M.

APPENDIXES.

A. *Logic and the Principle of Identity.*

T. L. G., p. 63: "‘Contrariwise,’ continued Tweedledee, ‘if it was so, it might be; and if it were so, it would be: but as it isn’t, it ain’t. That’s logic.’”⁴⁶

S. B., p. 159: The Professor said: "The day is the same length as anything that is the same length as *it*."

S. B., p. 161: Bruno observed that when the Other Professor lost himself, he should shout. "He’d be sure to hear hisself, ’cause he couldn’t be far off."

B. *Empirical Philosophers and Mathematics.*

T. L. G., p. 124: "‘. . . Now if you had the two eyes on the same side of the nose, for instance—or the mouth at the top—that would be *some* help.’

"‘It wouldn’t look nice,’ Alice objected. But Humpty-Dumpty only shut his eyes and said: ‘Wait till you’ve tried.’”

T. L. G., p. 112: "‘And if you take one from three hundred and sixty-five, what remains?’

"‘Three hundred and sixty-four, of course.’

"Humpty-Dumpty looked doubtful. ‘I’d rather see that done on paper,’ he said."

C. *Nominal Definition.*

T. L. G., p. 114: "‘When *I* use a word,’ Humpty-Dumpty said in rather a scornful tone, ‘it means just what I choose it to mean—neither more nor less.’

"‘The question is,’ said Alice, ‘whether you *can* make words mean different things.’

⁴⁶ Unfortunately, there is some doubt here as to whether Tweedledee, like Jevons, understood *is* to mean the same as ($=$), or, like Schröder, to mean the relation of subsumption. The first possibility alone would justify our contention. The next extracts illustrate the importance which the Professor and Bruno ascribed to the Principle of Identity.

“ ‘The question is,’ said Humpty-Dumpty, ‘which is to be master—that’s all.’ ”

D. Conformity of a Paradoxical Logic with Common Sense.

T. L. G., p. 162:

“But I was thinking of a plan
To dye one’s whiskers green,
And always use so large a fan
That they could not be seen.”
(Verse from White Knight’s song).

E. Idealists and the Laws of Logic.

T. L. G., p. 75: “ ‘...if he [the Red King] left off dreaming about you [Alice],’ [exclaimed Tweedledee], ‘where do you suppose you’d be?’ ”

“ ‘Where I am now, of course,’ said Alice.

“ ‘Not you!’ Tweedledee retorted contemptuously. ‘You’d be nowhere. Why, you’re only a sort of thing in his dream!’ ”

“ ‘If that there King was to wake,’ added Tweedledum, ‘you’d go out—bang!—just like a candle!’ ”

“ ‘I shouldn’t!’ Alice exclaimed indignantly. ‘Besides, if *I’m* only a sort of thing in his dream, what are *you*, I should like to know?’ ”

“ ‘Ditto,’ said Tweedledum.... ‘...you know very well you’re not real.’ ”

“ ‘*I am* real!’ said Alice, and began to cry.”

T. L. G., p. 157: “ ‘How *can* you go on talking so quickly, head downwards?’ Alice asked, as she dragged him out by the feet, and laid him in a heap on the bank.

“The Knight looked surprised at the question. ‘What does it matter where my body happens to be?’ he said. ‘My mind goes on working all the same. In fact, the more head downwards I am, the more I keep inventing new things.’ ”

T. L. G., p. 159: “‘. . . Everybody that hears me sing—either it brings the *tears* into their eyes, or else—’

“‘Or else what?’ said Alice, for the Knight had made a sudden pause.

“‘Or else it doesn’t, you know.’”

F. Distinction Between Sign and Signification.

T. L. G., pp. 159-160: “‘The name of the song is called “*Haddocks’ Eyes*.”’

“‘Oh, that’s the name of the song, is it?’ Alice said, trying to feel interested.

“‘No, you don’t understand,’ the Knight said, looking a little vexed. ‘That’s what the name [160] is *called*. The name really *is* “The Aged Aged Man.”’

“‘Then I ought to have said “That’s what the *song* is called,”’ Alice corrected herself.

“‘No, you oughtn’t: that’s another thing. The *name* is called “*Ways and Means*,” but that’s only what it’s *called*, you know!’

“‘Well, what *is* the song, then?’ said Alice, who was by this time completely bewildered.

“‘I was coming to that,’ the Knight said. ‘The song really *is* “A-sitting on a Gate”. . . .’”

G. The Principle of Permanence.

T. L. G., p. 172: “‘Can you do Subtraction?’ [asked the Red Queen], ‘Take nine from eight.’

“‘Nine from eight I ca’n’t, you know,’ Alice replied very readily: ‘but—’

“‘She ca’n’t do Subtraction,’ said the White Queen.”

H. Utility of Symbolic Logic.

A. A. W., pp. 121-122: “‘I quite agree with you,’ said the Duchess; ‘and the moral of that is—“Be what you would [122] seem to be”—or if you’d like it put more

simply—"Never imagine yourself not to be otherwise than what it might appear to others that what you were or might have been was not otherwise than what you had been would have appeared to them to be otherwise."'

" 'I think I should understand that better,' Alice said very politely, 'if I had it written down: but I'm afraid I ca'n't quite follow it as you say it.'

" 'That's nothing to what I could say if I chose,' the Duchess replied, in a pleased tone."

I. Universal and Particular Propositions.

T. L. G., p. 54: The Gnat had told Alice that the Bread-and-butter-fly lives on weak tea with cream in it; so:

" 'Supposing it couldn't find any?' she suggested.

" 'Then it would die, of course.'

" 'But that must happen very often,' Alice remarked thoughtfully.

" 'It always happens,' said the Gnat."

J. Denoting.

"*T. L. G.*, p. 59: Tweedledum and Tweedledee were, in many respects, indistinguishable, and Alice, walking along the road, noticed that "wherever the road divided there were sure to be two finger-posts pointing the same way, one marked 'TO TWEEDLEDUM'S HOUSE,' and the other 'TO THE HOUSE OF TWEEDLEDEE.'

" 'I do believe,' said Alice at last, 'that they live in the same house! . . . ' "

K. Non-Entity.

T. L. G., p. 137: " 'I always thought they [human children] were fabulous monsters!' said the Unicorn.

" 'Do you know,' [said Alice], 'I always thought Unicorns were fabulous monsters, too! I never saw one alive before!'

" [138] 'Well, now that we *have* seen each other,' said

the Unicorn, 'if you'll believe in me, I'll believe in you. Is that a bargain?' "

T. L. G., p. 127: " 'I see nobody on the road,' said Alice.

" 'I only wish *I* had such eyes,' the (White) King remarked in a fretful tone. 'To be able to see Nobody! And at that distance, too! Why, it's as much as *I* can do to see real people by this light!' "

A. A. W., p. 10: "And she [Alice] tried to fancy what the flame of a candle is like after it is blown out, for she could not remember ever having seen such a thing."

A. A. W., p. 84: "...this time it [the Cheshire Cat] vanished quite slowly, beginning with the end of the tail, and ending with the grin, which remained some time after the rest of it had gone.

" 'Well! I've often seen a cat without a grin,' thought Alice; 'but a grin without a cat! It's the most curious thing I ever saw in all my life!' "

A. A. W., pp. 98-99: "...the Dormouse went on. . . . ; 'and they drew all manner of things—everything that begins with an M—'

" 'Why with an M?' said Alice.

" 'Why not?' said the March Hare.

"Alice was silent.

"...[the Dormouse] went on: '—that begins with an M, such as mouse-trap, and the moon, and memory, and muchness, you know you say things are "much of muchness"...did you ever see such a thing as a drawing of a muchness?'⁴⁷

[99] " 'Really, now you ask me,' said Alice very much confused, 'I don't think—'

" 'Then you shouldn't talk,' said the Hatter."

⁴⁷ This extract also illustrates the chapter on "Denoting."

L. Objects of Mathematical Logic.

T. L. G., p. 149: " 'I was wondering what the mouse-trap [fastened to the White Knight's saddle] was for,' said Alice. 'It isn't very likely there would be any mice on the horse's back.'

" 'Not very likely, perhaps, said the Knight, 'but, if they *do* come, I don't choose to have them running all about.'

" 'You see,' he went on after a pause, 'it's as well to be provided for *everything*. That's the reason the horse has anklets round his feet.'

" 'But what are they for?' Alice asked in a tone of great curiosity.

" 'To guard against the bites of sharks,' the Knight replied."

M. Gedankenexperimente.

T. L. G., p. 92: "Alice laughed. 'There's no use trying,' she said: 'one *ca'n't* believe impossible things.'

" 'I daresay you haven't had much practice,' said the [White] Queen. 'When I was your age, I always did it for half-an-hour a day. Why, sometimes I've believed as many as six impossible things before breakfast.'"

A. A. W., p. 11: "She [Alice] generally gave herself very good advice (though she very seldom followed it), and sometimes she scolded herself so severely as to bring tears into her eyes; and once she remembered trying to box her own ears for having cheated herself in a game of croquet she was playing against herself, for this curious child was very fond of pretending to be two people."

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